A Unified Approach for Modeling Complex Occlusions in Fluid Simulations

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1 Introduction

A few recent papers, [Enright 2002; Foster 2001; Foster 1996], have described the development of a fluid simulation method useful for computer graphics. We have implemented a system based on these papers, and propose an extension that augments and simplifies the handling of occlusions (impermeable solid objects). In the referenced papers, occlusions are treated differently based on whether or not they are moving, and discussion of how to deal with complex occlusions efficiently is minimal. The technique we have developed unifies the treatment of static and dynamic objects, and is able to better represent fluid-occlusion interactions at the low simulation resolutions used.

2 Unified Occlusion Representation

In order to unify all of the occlusions affecting the simulation, we represent their geometry using a level set instead of relying on a polygon mesh directly (as was done in [Enright 2002; Foster 2001; Foster 1996].) A level set alone can only capture the instantaneous geometry of the occlusions, thus it cannot model the effects of the occlusion velocities and slip conditions. We store this information in two addition fields – a vector field for the velocities and a scalar field for the slip conditions. The unified occlusions representation, $\psi(\mathbf{x})$, is thus composed of a level set, $\phi(\mathbf{x})$, occlusion velocity field, $\mathbf{u}(\mathbf{x})$, and slip condition field, $\alpha(\mathbf{x})$ such that:

$$\psi(\mathbf{x}) = \{\phi(\mathbf{x}), \mathbf{u}(\mathbf{x}), \alpha(\mathbf{x})\}$$

The global occlusions representation is the union of all the individual occlusions. For our purposes the union operator is defined as:

$$\psi_{A\cup B}(\mathbf{x}) = \begin{cases} \psi_A(\mathbf{x}) & \text{if } \phi_A(\mathbf{x}) \le \phi_B(\mathbf{x}) \\ \psi_B(\mathbf{x}) & \text{otherwise} \end{cases}$$

This union operator, since it is both associative and distributive, allows considerable flexibility in the calculation and partial caching of the global occlusions model.

3 Creating Occlusions Representations

The three fields that represent occlusions in our approach often need to be generated from explicit geometry representations. The following method is used to compute the set of fields for one occlusion.

3.1 Level Set. Of the three fields, the generation of the level set is the most complex. Our method, which is both efficient and robust, consists of the following steps: (1) Explicit geometry is converted into a structure that allows for efficient calculation of ray intersections. (2) Rays are cast along a number of axis directions, recording data indicating whether the center of a voxel is inside or outside, and an initial distance from the surface. This data is then resolved into a rough distance function. (3) Discontinuities near the zero-level set (which can result from imperfections such as gaps in the geometry) are removed. (4) The level set is then "smoothed" via a standard re-initialization algorithm.

3.2 Velocity Field. Our calculation of the velocity field is straightforward. Each occlusion is associated with a motion path expressible as a function mapping time t to a transformation matrix, $\mathbf{T}(t)$. The velocity field is obtained by taking the product of the position and the component-wise derivative of this motion path: $\mathbf{u}(\mathbf{x},t) = (\mathbf{T}'(t))\mathbf{x}$. In the case of a deforming object, a more sophisticated approach must be taken.

3.3 Slip Field. We use a single value for the slip condition of each occlusion, so the slip field is a constant.

4 Incorporation into Fluid Simulation

The occlusions are relevant to two major aspects of the fluid solver, the mass conservation calculation and the advection velocity field.

4.1 Mass Conservation. Incompressibility is enforced by solving the equation $\nabla\cdot\mathbf{u} = \nabla^2\mathbf{p}$ for pressure, $\mathbf{p}$. During the calculation of $\nabla\cdot\mathbf{u}$, vector occlusion velocities are used instead of fluid velocities in regions interior to the occlusions. The occlusions are incorporated into the Poisson matrix (which represents the Laplacian operator) by omitting cells that lie completely within the occlusions, and using von Neumann boundary conditions for the remaining faces whose centers' lie within the occlusions.

4.2 Advection Velocity Field. The advection velocity field, which is used to advect both the level set and the fluid velocity field, must accurately account for the occlusions. This is achieved via our new method of *constrained velocity extrapolation*. This is similar to the method described in [Enright 2002] for extrapolating fluid velocities into the air, the main distinction however, besides the fact that we are extrapolating into occlusions rather than the air, is that constraints are imposed on the velocities after extrapolation. These constraints reflect the effects of three space varying aspects of the occlusions: *the slip coefficient*, *the surface orientation*, and *the velocity*. It is convenient to calculate the resulting constrained velocity as the sum of two orthogonal components – one parallel to the occlusion normal and the other perpendicular:

$$\mathbf{u}_{\parallel} = \begin{cases} \operatorname{proj}_{\mathbf{n}_o}\mathbf{u}_f & \mathbf{n}_o\cdot\mathbf{u}_f \ge \mathbf{n}_o\cdot\mathbf{u}_o \\ \operatorname{proj}_{\mathbf{n}_o}\mathbf{u}_o & \textit{otherwise} \end{cases} \qquad \mathbf{u}_{\perp} = \operatorname{ortho}_{\mathbf{n}_o}\left(\mathbf{u}_f(1-2\alpha_o) + \mathbf{u}_o\alpha_o\right)$$

Where $\mathbf{u}_f$ is the unconstrained extrapolated fluid velocity, $\mathbf{u}_o$ is the occlusion velocity, α_o is the occlusion slip coefficient, and $\mathbf{n}_o$ is the normalized gradient of the occlusions level set. The slip coefficient parameterizes the range of boundary conditions from full-slip to no-slip along the range [0,1]. (It should be noted that while we do here suggest a full parameterization of the slip coefficient we have only thoroughly tested the full-slip extreme were α=0.)

5 Results

The results are promising. Our unified approach to modeling occlusions has significantly decreased the difficulty of specifying complex dynamic scenes. Incorporating the occlusions into the fluid solver via *constrained velocity extrapolation*, because it models the subtleties of occlusions, result in more realistic and precise fluid behavior in the presence of objects.

Figure 1, courtesy of Jason Cobill, illustrates the accuracy at which occlusions, in this case a scan of a human head, can be incorporated into the fluid simulation. Figure 2, courtesy of Conrad Dueck, is a still from an animation consists of a series of cups scooping up fluid from a rectangular container. Figure 3, courtesy of Chris Pember, shows the rendering of a simple scene via photon mapping.

Figure 1

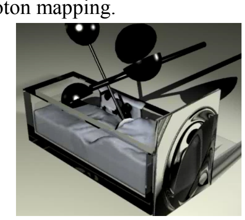

Figure 2

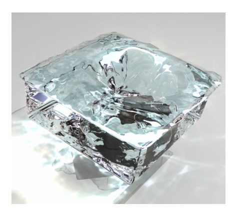

Figure 3

References

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